

Cardinality of Power Set: Interpreting Formula

- Calculate by considering subsets of various cardinalities.
- Calculate by considering whether a member should be included.

Relation : set of ^{tuples.} ordered pairs

e.g. a relation on $\boxed{\{1, 2, 3\}}$ ^S and $\boxed{\{a, b\}}$ ^T

- Is $(1, a)$ a relation on S and T?
No! $\because (1, a)$ is not a set.

Set of Possible Relations

- **Set** of possible relations on S and T:
- Dedicated symbol for **set** of possible relations on S and T:
- Declare that set r is a relation on S and T:

Example: Enumerate all relations on {a, b} and {2, 4}.

Hint: How many?

Relational Operations: Domain, Range, Inverse

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

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Exercise: Relate the domains and ranges of r and its inverse.

Relational Operations: Image

$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$

Exercises

- **Image** of $\{a, b\}$ on r ?
- **Image** of $\{1, 2\}$ on r ?
- **Image** of $\{1, 2\}$ on the **inverse** of r ?
- Calculate r 's **range** via an **image**.
- Calculate r 's **domain** via an **image**.

Relational Operations: Restrictions vs. Subtractions

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$
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$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

Relational Operations: Overriding

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

Example: Calculate r overridden with $\{(a, 3), (c, 4)\}$

Hint: Decompose results to those in t 's domain and those not in t 's domain.